Physics Qualifying Exam UCI October 4, 1972

QUANTUM MECHANICS I

1. Consider a 2 state quantum mechanical system whose Hamiltonian is H_1 and whose eigenstates are $\psi, \bar{\psi}$. At time T_o H_1 is turned off and $H_2 = H_{2_o} + \Delta H_2 \neq H_1$ is turned on. The eigenstates of H_{2_o}, ψ_1, ψ_2 (linear combinations of $\psi, \bar{\psi}$) have energies E_1, E_2 and decay with time constants Γ_1, Γ_2 (due to ΔH_2). We observe this system at some time $t > T_o$. Compute the probability for finding ψ and $\bar{\psi}$ at time t, if at $t = T_o$ the particle is in the state ψ.

2. A particle moves in one dimension with energy $E > 0$, in the presence of the potential $V(x) = -\lambda\delta(x)$.

   a. Find an exact expression for the scattering amplitude, and the probability that the particle is reflected from the potential.

   b. From the properties of the scattering amplitude in (a), deduce the energies of any bound states that may be associated with the potential.

3. A particle with spin $\frac{1}{2}$ is placed in a D.C. magnetic field $\hat{z}\,H_o$ along z with its spin up (in the + z direction). At $t = 0$, a constant D.C. magnetic field of strength h is applied parallel to the x axis, and the field h remains on for all times $t > 0$. Find an explicit expression for the wave function of the spin for times $t > 0$.

4. A particle of mass M moves in the harmonic oscillator potential $\frac{1}{2} kr^2$ in three dimensions. An electric field of strength E_o is applied parallel to the z axis, and the particle has charge e.

   a. In the presence of the field, find the form of the eigenfunctions and eigenvalues.

   b. Suppose the particle is in the ground state. By comparing $\langle \vec{r} \rangle$ in the presence and absence of the field, find the magnitude of the electric dipole moment induced by the field.

5. A particle is governed by the harmonic oscillator Hamiltonian

$$H = \frac{1}{2} p^2 + \frac{1}{2} \Omega_o^2 x^2$$

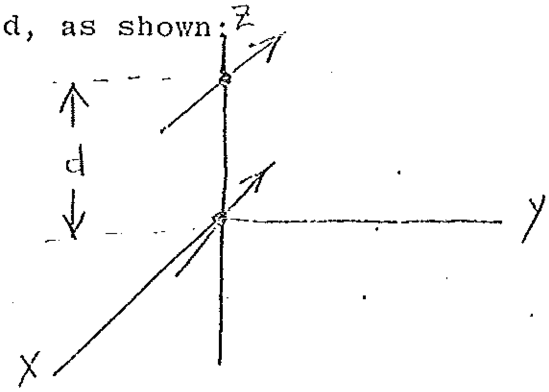
   a. If x(t) is the position operator in the Heisenberg representation, find an explicit form for x(t) in terms of operators in the Schrödinger representation.

   b. At time t = 0, the particle is placed in the state

$$\varphi(x) = \frac{1}{\sqrt{\pi^{\frac{1}{2}} \Delta}} \exp[-(x-x_o)^2/2\Delta^2] \quad .$$

   If $\langle x \rangle$ is the expectation value of the position operator, find an expression for $\langle x \rangle$ as a function of time. (Note that the wave function $\varphi(x)$ is properly normalized in the statement above.)

6. Two fermions with spin $\frac{1}{2}$ are placed along the z axis separated by the distance d, as shown:



They are described by the Hamiltonian

$$H = -H_o(\sigma_1^z + \sigma_2^z) + J\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + K(\vec{\sigma}_1\cdot\vec{\sigma}_2)^2$$

where J is positive and considerably larger than either H_o or K.

a. Find the energy levels and eigenfunctions of H and draw an energy level diagram, indicating which eigenfunction is associated with each energy level.

b. The fermions are in the ground state, in the energy level scheme of (a). A magnetic field of frequency Ω and wave vector q propagates parallel to the z axis:

$$\mathfrak{H}(\vec{x},t) = h_o(\hat{x} + i\hat{y})e^{iqz}\,e^{-i\Omega t} \quad .$$

Indicate which transition in the diagram of part (a) can be induced by a field of this form, and calculate the matrix element required for a computation of the transition probability.

c. If the two fermions are the protons in an H_2 molecule, and if the field is a microwave field, would you expect the transition considered in (b) to be strong or weak, compared to the NMR signal of an isolated proton? You should not have to do any calculation to answer this, and one sentence will suffice.

7. An electron is placed initially into an eigenstate $\varphi_o(\vec{x})$ of the Hamiltonian H_o. The electron is subjected to a weak electric field $E_o(t)$ applied parallel to the x axis with the time dependence

$$E_o(t) = \begin{cases} E_o & -\frac{T}{2} < t < +\frac{T}{2} \\ 0 & \text{otherwise} \end{cases} \quad .$$

a. Find an expression for the probability that the electron is found in the state $\varphi_n(\vec{x})$ after the time $+ T/2$. You may assume the field E_o is weak, and that all eigenstates of H_o are non-degenerate.

b. Imagine this result applied to a hydrogen atom. What is required of E_o and T for perturbation theory to be valid? You should be able to answer this from the structure of the answer to (a), without detailed calculation.

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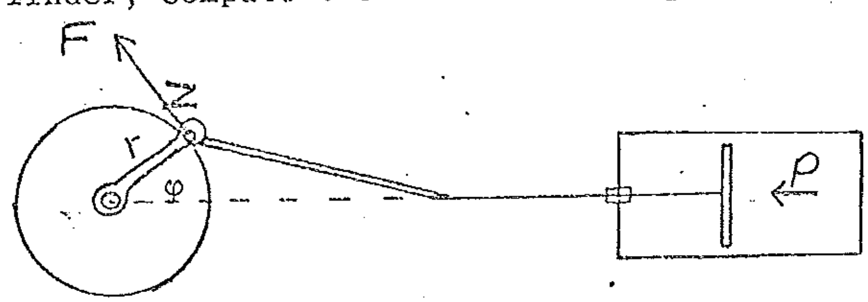
CLASSICAL MECHANICS

1. Consider a river which flows from North to South. Compute the angle which the normal to the water surface makes with a plumb bob.

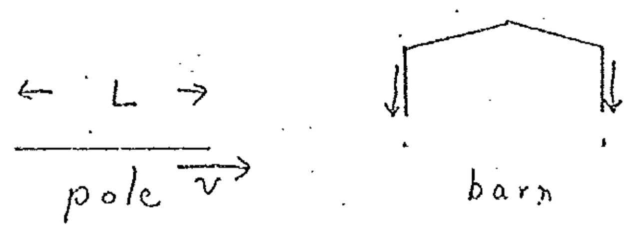
2. Consider the scattering of point particles of mass m from a perfectly hard sphere of radius R and mass M. Compute the differential and total cross section in the center of mass frame. Compute the total cross section in the laboratory frame.

3. Consider the drive mechanism of the steam engine illustrated in the sketch below. If P is the steam pressure in the cylinder, compute the force F developed at the crank pin Z.



4. Do one of the two parts only. Explain carefully and quantitatively.

a. Observer A sits in the rest frame of a barn L meters long with doors at either end, which are open. A pole vaulter B carries a pole of length L at velocity v, and runs into the barn.
Observer A has the barn doors closed when both B and the pole are in the barn, thus containing B.



4. (cont'd.)

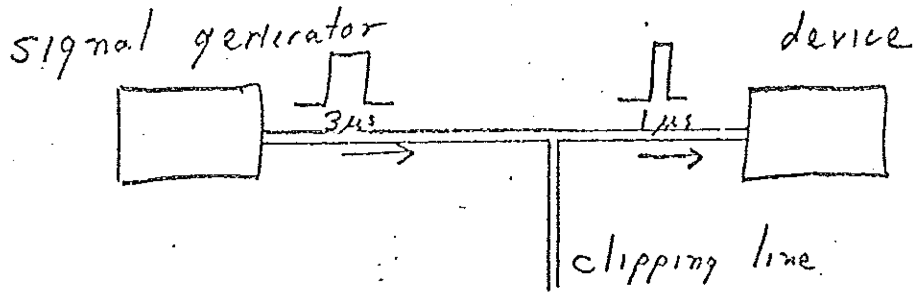
On the other hand, B sees the length of the barn to be less than L, since it is foreshortened by a factor γ. He knows that if the barn doors are closed simultaneously, one door or the other would strike the pole. What is the resolution of this apparent difference of opinion?

b. Explain the twin paradox. Twin A remains on earth, and twin B travels 1.5×10^{12} meters at the velocity c/2, then stops and returns to earth at the same speed. Which twin is older, and by how much? Why is this problem called a paradox, and why is it not? Discuss the aging of each twin at each stage of the trip.

5. A particle moves in a circular orbit under a central force F(r) directed from the center of the circle. Obtain an equation of motion for small radial displacements of the particle from the circular orbit. Obtain a condition on F(r) and dF(r)/dr for stable oscillations about circular orbits. Determine which power laws permit stable orbits.

6. a. Write down the wave equation for a string of linear mass density ρ and tension T. Consider an infinitely long continuous string which, for $X < 0$ has linear mass density ρ, but has a density $\rho_2 > \rho_1$, for $X > 0$. A wave train oscillating with frequency ω is incident from the left. Find the reflected and transmitted intensities and phases.

b. Solve the following experimental problem. A physicist wishes to pulse a piece of equipment which is capable of responding to a one μsec. long positive square pulse. However he has a signal generator which is capable only of producing a 3 μsec. long square pulse. He attempts to use a "clipping line".



How should he choose the parameters of this setup to obtain the desired result. (He uses transmission lines of propagation velocity c/2.)

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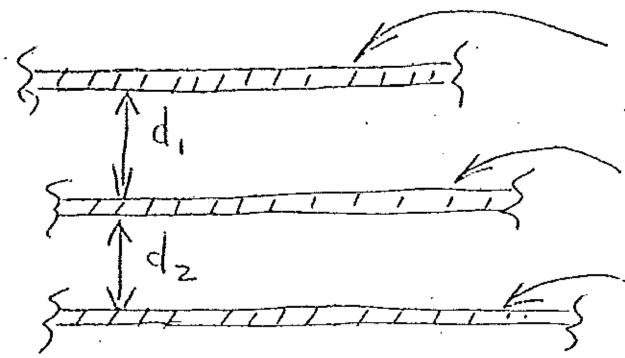
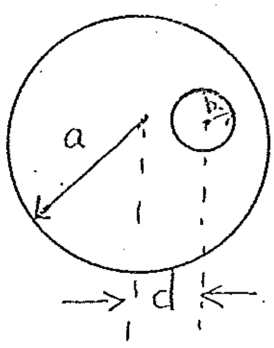
ELECTRICITY AND MAGNETISM SECTIONS

1. Three metal plates, each of large area A, carry total charge Q_1, Q_2, and Q_3, as indicated in the sketch below. Find the total charge on each <u>side</u> of the three plates.



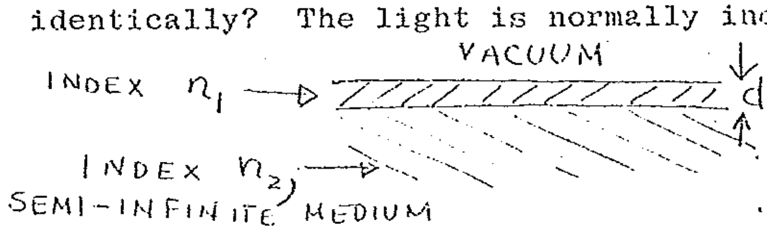
2. Consider a wire in the following shape:



The wire has a circular cross section of radius a, with an off-center hole of radius b < a as shown.

The wire carries a current in the form of a uniform current density J. Find an expression for the magnetic field in the hole. Draw a sketch of the lines of H in the hole. (Hint: Think of the superposition principle.)

3. Light of frequency ω falls on the layered dielectric structure illustrated in the sketch below. How thick should the layer of index of refraction n_1 be for the reflectivity to vanish identically? The light is normally incident on the structure.



4. A particle of charge Q is placed at rest at the origin of the coordinate system at the time $t = 0$. A magnetic field H_o is present parallel to the z axis, and an electric field of strength E_o is parallel to the x axis. Solve for the motion of the particle when $t > 0$.

5. A charge q of mass m and charge q is subject to a Newtonian frictional force

$$= - \gamma \vec{v},$$

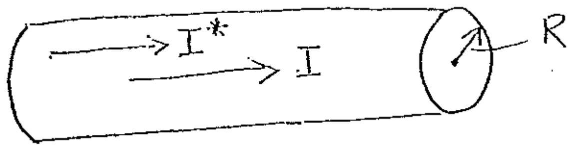
where γ = constant, $\vec{v}$ = instantaneous velocity. Upon it is incident a circularly polarized electromagnetic plane wave of intensity I and angular frequency ω in the z direction.

   a. Calculate (in the steady state) the rate at which energy is absorbed from the wave by the particle.

   b. Calculate directly (i.e., by use of the Lorentz force equation) the rate at which angular momentum is absorbed.

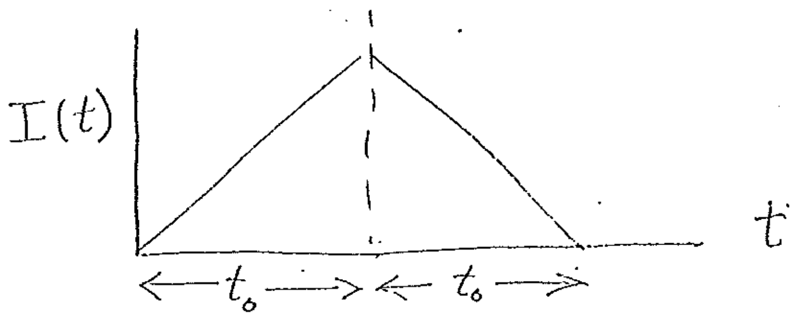
6. A current I(t) with uniformly distributed current density is passing through a very long cylinder of radius R, along the axis. The cylinder contains a gas which suddenly at some arbitrary time T becomes highly ionized, so that it acquires a large conductivity σ. After T no net charge exists inside the cylinder, and only the axial field E_z remains. The ionized gas obeys Ohm's law, $j^* = \sigma E_z$. Assume the

current density in the gas, j^*, is uniform to the wall and neglect any special effects from this wall.



a. what simple equation relates $I(t)$ to the current in the ionized gas, I^*, for $t > T$?

b. Assume $I(t)$ is triangular in time, i.e.,



T may fall anywhere in the lifetime of $I(t)$. Consider two cases, $T = t_o/10$ and $T = t_o$. Solve your equation for the total current, $I + I^*$, and sketch the solution. If time is pressing, use your intuition to sketch the result without solving the equation in detail.

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QUANTUM MECHANICS II

1. a. Consider the bound state of an electron and a positron (positronium). Give an energy level diagram of this object through $n = 2$, labeling each state spectroscopically, giving the charge conjugation quantum number and parity.

   b. For $n = 1$, give the fine structure and Zeeman effect splittings by using an $A\ \vec{S}_1 \cdot \vec{S}_2$ interaction between the electron and positron.

2. Consider the helium atom. Estimate the ground state energy either by using perturbation theory or by the variational method.

   Potentially useful integrals and formulas.

$$\int d^3r\ e^{-2kr} = \frac{\pi}{k^3}$$

$$\int d^3r\ \frac{e^{-2kr}}{r} = \frac{\pi}{k^2}$$

$$\int d^3r_1 d^3r_2\ \frac{e^{-2k(r_1 + r_2)}}{|\vec{r}_1 - \vec{r}_2|} = \frac{5}{8}\frac{\pi^2}{k^5},$$

$$\nabla_r^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right)$$

3. a. In atomic physics, the spin orbit interaction has the form $A\ \vec{\ell} \cdot \vec{s}$. Derive an expression for the coefficient A in a hydrogenic atom. Don't worry about factors of two, feel free to use qualitative arguments, but get the dependence on the relevant variables correct. Estimate the magnitude of the fine structure splitting in the optical spectrum of sodium.

   b. An electron in a state of orbital angular momentum ℓ is placed in a weak magnetic field. Find an expression for the g-factor.

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THERMODYNAMICS AND STATISTICAL MECHANICS

1. A magnetic material is placed in an external magnetic field H. The external field is changed in an adiabatic manner by the amount ΔH.

   a. Show that the change in temperature of the system is given by

$$\Delta T = -\frac{T}{C_H(T)} \left(\frac{\partial M}{\partial T}\right)_H \Delta H ,$$

   where M is the magnetization, and $C_H(T)$ the specific heat in constant field. (Hint: Consider the use of the Maxwell relations for the magnetic system).

   b. For a paramagnetic system, $M = \chi H$. From what you know of the temperature dependence of χ for paramagnetic systems, do you expect $\Delta H > 0$? Do not just state the answer, but explain briefly.

2. A gas is described by the equation of state

$$P(V-b) = RT \exp\left[-\frac{a}{RTV}\right] ,$$

   where R is the gas constant. Sketch the form of the isotherms in the P-V plane, discuss briefly the behavior of the system in the interesting regions of the sketch, and find an expression for any critical points that may be present.

3. A set of N independent spins are placed in an external magnetic field H. The spins are at temperature T.

a. Find an expression for the entropy as a function of magnetic field and temperature.

b. Draw a sketch of the entropy as a function of magnetic field, for fixed temperature. Explain the physical origin of the value of the entropy as $H \to 0$, $H \to \infty$.

4. The electron density in a white dwarf star is 10^{30} electrons/CM^3, and the temperature of the star is 10^7K. The electrons in the star move with speeds near the velocity of light so the energy of an electron is $\epsilon = cp$, where p is the electron's momentum. Assuming that this relation is valid for all p, compute the electron pressure P in the star. (Hint: the electrons form a degenerate Fermi gas at these temperatures and pressures.)

5. Two infinite parallel plates possess black-body surfaces, and are maintained at temperatures T_1 and T_2 respectively. Calculate the rate $q_{12}(\omega)$ at which energy of frequency ω is transferred from one surface to the other per unit area and unit time. Calculate the total rate of energy transfer when T_1 is close to T_2.

6. By the methods of statistical mechanics, show that the entropy of the ideal gas is given by

$$S = Nk\, So + Nk \ln(VT^{3/2}).$$

Exhibit an explicit expression for the constant So.

PHYSICS QUALIFYING EXAM UCI October 6, 1972

MATHEMATICAL PHYSICS

1. Minimize the function

$$f(x,y,z) = x^2 + 3xy + 2y^2 + 4yz + z^2$$

subject to the constraints

$$x + y = 1$$

$$zy + z = 1$$

2. Evaluate the integral

$$I(\alpha) = \text{P.V.} \int_0^\infty \frac{x^\alpha \, dx}{x^2 - 1} \quad , \qquad 0 < \alpha < 1 \quad .$$

3. Solve the homogeneous integral equation

$$\phi(x) = \lambda \int_{-\pi}^{+\pi} \cos(x-y)\, \phi(y)\, dy \quad .$$

4. Using the saddle point method, obtain the leading term in the asymptotic behavior of the function $I(k)$:

$$I(k) = \int_c e^{-t} \left(\frac{t + \frac{i}{2}}{t - \frac{i}{2}} \right)^{i \frac{k^2}{4}} dt \quad .$$

5. Use the WKB approximation to discuss the energy levels of a particle of mass m moving in a one dimensional potential $V(x) = \frac{1}{2} k|x|^n$, where $n > 0$. In particular, what are the energy levels in the limit $n \to \infty$? Why is your answer reasonable?

6. Radioactive gas atoms are introduced into a non-radioactive background of the same gas, at the same temperature and pressure. The radioactive particles are introduced at $x = 0$ and $t = 0$ into a very long, narrow box, so a one dimensional diffusion process occurs.

   a. The box walls are infinitely far away, and the probability distribution of particle position satisfies

$$\frac{\partial P(x,t)}{\partial t} = D\,\frac{\partial^2 P(x,t)}{\partial x^2}$$

   b. Now suppose one wall of the box is brought to $x = L$, the other wall still at infinity. This wall at L absorbs all radioactive gas particles which reach it. What is $P(x,t)$ now? State your reasoning.

7. Determine the behavior of the function

$$\Gamma(Z) = \int_0^\infty e^{-t}\, t^{z-1}\, dt \quad , \qquad z > 0$$

in the limit as $z \to 0$.

PHYSICS QUALIFYING EXAM UCI October 6, 1972

GENERAL PHYSICS SECTION

1. Order the following gases in order of increasing index of refraction:

$$S,\ HC\ell,\ He,\ CH_4,\ CH_3OH$$

2. What is the minimum energy in the center of mass frame necessary to produce a proton-anti proton pair? Derive the result.

3. Determine which of the following decays can proceed, and specify which interaction. If the decay cannot occur, explain the reason.

$$\Lambda^o \rightarrow p + e^- + \bar{\nu}_e$$

$$\pi^o \rightarrow \gamma + \gamma + \gamma$$

$$\Delta^+ \rightarrow n + \pi^+$$

$$K_L^o \rightarrow \pi^+ + \pi^-$$

4. (a) Can one do X-ray astronomy on the earth's surface? If not, explain why. If one can, explain briefly what kind of apparatus is required.

   (b) There has been a considerable amount of interest in the possibility of observing solar neutrinos. Why would this be interesting?

5. What wavelength radiation is required to make a photoelectric cell using potassium? Explain the reason for your answer.

6. In the late nineteenth century, the notions of quantum theory were developed in only primitive form, of course. As a consequence, physcists faced many puzzles. One of these was the very large electrical conductivity observed in metals

6. (cont'd.)
at low temperature (think of normal metals, not superconductors). Why was this a problem in those days, and what idea cleared up the problem?

7. At one time, physicists thought that electrons might be present inside the nucleus. From what you know of the properties of particles and their interactions, give one or more reasons why this possibility is unreasonable. Include order of magnitude estimates of the relevant physical quantities that enter into your considerations.

8. (a) Theoretical physicists postulated the existence of the neutrino many years before its existence was confirmed by experimentalists (Reines-Cowan experiment). What observation prompted Pauli to propose the existence of the neutrino?

   (b) It is now realized that there are two distinct classes of neutrinos in nature. Briefly describe them, and in particular indicate the kinds of interactions in which each class of neutrinos participates.

9. On a clear night, signals from radio stations can be heard many thousands of miles from the source. On the other hand, television signals from transmitters comparable in power to radio transmitters can be received within only a few tens of miles of the antenna. Why? (Compare television signals with AM radio signals.)

10. The electrical conductivity of matter is a remarkable property, in that it may vary by as much as seventeen orders of magnitude, if one compares the conductivity of wide classes of materials with each other. Briefly and clearly explain the physical origin of the difference between metals,

10. (cont'd.)
insulators, and semiconductors. Crudely sketch the temperature dependence of the conductivity of

(a) a metal

(b) a semiconductor.

11. Below are listed (vertically) conservation laws, and horizontally types of particle interactions. Which laws are valid for which interactions? Put an X in the appropriate box if the law is invalid.

Parity	Strong	Electromagnetic	Weak
Charge Conjugation			
Baryon Conservation			
Lepton Conservation			
Time Reversal			
TCP			
CP			
Isotopic Spin			
Strangeness			

12. What is the approximate radius of

(1) proton
(2) electron
(3) Oxygen atom
(4) Earth
(5) Earth's orbit
(6) particle in fog
(7) Galaxy

13. Provide approximate estimates of

1. Surface temperature of sun.
2. Highest steady magnetic field which can be reached with superconducting magnets.
3. Earth's magnetic field at 5 earth radii.
4. Velocity of 100 Kev electron

13. (cont'd.)

    5. Lowest temperature currently attained.
    6. Scaling length (e - folding length) of Earth's atmosphere.
    7. Age of earth.
    8. Half life of neutron.

14. Plasmas exist in both metals and as hot, ionized gases. Describe briefly how these two types of plasmas differ, how the relevant parameters vary in magnitude and what sort of phenomena might be seen in one case, but not the other.

15. In the famous elevator gedanken-experiment with which Einstein illustrated the principle of equivalence, an observer could not observe a gravitational field <u>if</u> the elevator was freely falling. However, the earth is freely falling in the moon's gravitational field. Explain the tides.

16. A dust particle is placed inside a long pipe filled with air. If the particle strikes the walls it is simply reflected, and thus can move only along the pipe. Ten seconds later, the experimenter finds that the dust particle has moved one centimeter to the left. How long will it take the particle to move a meter from that position?